**Solution of a Quadratic Equation By Completing The Square**

 **Working Rule :-** To find the solution of the quadratic equation by using Completing the Square method following steps are followed :

**Step 1:-** Write the quadratic equation in standard form i.e. ax2 + bx + c = 0

**Step 2:-** Divide the whole equation by the coefficient of x2 we get the equation as

 **x2 +** $\frac{b}{a}$ **x +** $\frac{c}{a }$ **= 0**

**Step 3:-** Shift the constant term $\frac{c}{a}$ on RHS we get  **x2 +** $\frac{b}{a}$ **x = -** $\frac{c}{a}$

**Step 4:-** Add square of half of the coefficient of x on both side to obtain (a + b)2 or (a – b)2 format depending upon the sign of the coefficient of x, we get

 **x2 +** $\frac{b}{a}$ **x + (**$\frac{b}{2a})$**2 = (**$\frac{b}{2a})$**2 -** $\frac{c}{a}$

**Step 5:-** Write LHS as perfect square and simplify RHS than we get

 **( x +** $\frac{b}{2a}$ **)2 =** $\frac{b^{2}- 4ac}{4a^{2}}$

shifting the constant term on RHS i.e. **x = -** $\frac{b}{2a}$ **±** $\frac{√b^{2}- 4ac}{2a}$ **=** $\frac{-b \pm √b^{2}- 4ac}{2a}$

**Example : find the roots of the quadratic equation 2x2 – 5x + 3 = 0 , by using the method of Completing The Square.**

Sol: As the equation is in standard form. Now divide whole equation by the coefficient of x2 i.e 2 we get

 $\frac{2 x^{2}}{2}$ **-** $\frac{5}{2}$ **x +** $\frac{3}{2}$ **= 0**

* **x2 -** $\frac{5}{2}$ **x +** $\frac{3}{2}$ **= 0**
* **x2-** $\frac{5}{2}$ **x = -** $\frac{3}{2}$ (shifting the constant term on RHS)

adding square of half of the coefficient of x on both side

* **x2 -** $\frac{5}{2}$ **x + (** $\frac{5}{4} )$**2 =-** $\frac{3}{2}$ **+ (** $\frac{5}{4} )$**2**
* **( x -** $\frac{5}{4}$ **)2 =** $\frac{25}{16} $**-** $\frac{3}{2} $ **=** $\frac{1}{16}$
* **x -** $\frac{5}{4}$ **= ±** $\frac{1}{4}$
* **x =** $\frac{5}{4}$ **±** $\frac{1}{4} $, on solving we get the two values of x as
* **x =** $\frac{5}{4} $**+** $\frac{1}{4}$ **=** $\frac{6}{4}$ **=** $\frac{3}{2}$ **and x =** $\frac{5}{4} $**-** $\frac{1}{4}$ **=** $\frac{4}{4}$ **= 1**
* we get the values of x are **3/2** and **1**

**Some Important Questions From Chapter 4**

**Q1. Represent the following situations in the form of quadratic equations:**

1. The area of a right angled triangle is 600 cm2. The base of the triangle exceeds the altitude by 10 cm. We would like to find the altitude of the triangle.
2. The sum of two numbers is 18. The sum of their reciprocal is 1/4 .We would like to find the numbers.
3. The product of Rahul’s age (in years) five years ago and the age ten years later is 16. We would like to find the present age of Rahul.
4. A train travels a distance of 300 km at a constant speed. If the speed of the train is increased by 5 km/hr. , the journey would take 2 hrs. Less. We would like to find the speed of the train.
5. A motorboat whose speed in still water is 18 km/hr., takes 1 hr more to go 24 km upstream then to return downstream to the same spot. We would like to find the speed of the stream.
6. A train travelling with a constant speed for 360 km, would have taken 48 minutes less to travel the same distance were 5 km/hr. More. We would like to find the speed of the train.

**Solution : The above mentioned problems are to be solved only for making quadratic equations :**

1. Let the altitude of the triangle = x

Then A.T.Q.

Base of the triangle = x + 10

Now area of the triangle = 600

* $\frac{1}{2}$ x . (x + 5) = 600 (area of triangle = $\frac{1}{2}$ base x altitude)
* x.(x + 5) = 1200
* x2 + 5x -1200 = 0 is the required quadratic equation.
1. Let one number = x
* Second number = 18 – x ( as the sum of two numbers is 18)

Now A.T.Q.

 Sum of reciprocals = $\frac{1}{4}$

* $\frac{1}{x}$ + $\frac{1}{18-x}$ = $\frac{1}{4}$
* $\frac{18-x+x }{x(18-x)}$ = $\frac{1}{4}$
* $\frac{18}{18x-x^{2}}$ = $\frac{1}{4}$
* 18x – x2 = 72
* x2 – 18x + 72 = 0 is the required equation.
1. In the questions of ages always remember take variable for present age. Because all the ages (before or later or after) can be calculated by knowing present age . This is the hint for Q1. (iii).
2. Let the speed of the train = x km/hr.

Now if distance is 300 km then

Time taken (t1) = $\frac{300}{x}$ hr. ( time = distance / speed)

We have given that, if speed increased by 5 km/hr it takes 5 hr less

* $\frac{300}{x+ 5}$ = t1 – 5
* $\frac{300}{x+ 5}$ = $\frac{300}{x}$ - 5
* $\frac{300}{x+5}$ - $\frac{300}{x}$ = -5
* $\frac{300x-300 ( x+5)}{x ( x+5)}$ = -5
* $\frac{300 x-300 x-1500}{x^{2}+ 5x }$ = -5
* -1500 = -5x2 – 25x
* 5x2 + 25x – 1500 = 0 is the required quadratic equation.
1. In these type of questions we have to assume the speed of stream, then we can find upstream and downstream speed .

Let the speed of stream = x km/hr. And the speed in still water = 18 km/hr.

* Speed of boat downstream = (18 + x) km/ hr.

 And upstream speed of boat = (18-x) km/hr

Now A.T.Q. for the distance = 24 km we have

 $\frac{24}{18-x} $ = $\frac{24}{18+x}$ +1 => $\frac{24}{18-x}$ - $\frac{24}{18+x}$ = 1

* $ $ $\frac{24\left(18+x\right)- 24(18-x)}{\left(18-x\right)\left(18+x\right)}$ = 1
* $\frac{472+24x-472+24x }{324- x^{2}}$ = 1
* 48x = 324 – x2

1. Solve the question same as Q1. (iv).

**Q2. Check whether the following are quadratic equations or not:**

1. (x + 1)2 = 2(x – 3) (vi) (x - 2) (x +1) = (x – 1) ( x + 3)
2. (x – 3) (2x + 1) = x ( x + 5) (vii) (2x – 1) ( x – 3) = (x +5) ( x – 1)
3. (x + 2)3 = 2x ( x2 – 1) (viii) x3 – 4x2 – x + 1 = ( x – 2)3
4. (x + 6) ( x + 4) = x ( x +2) + 7 (ix) x3 – x2 = ( x – 1 )3
5. 2( x-1)2 = 4x2 – 2x +1 (x) (x2 + 2x)2 = x4 + 3 + 4x2

Solution. In these question we will simplify the equations and if we got the standard form i.e **ax2 + bx + c = 0,** of quadratic equation then it is quadratic equation otherwise not.

(ii) (x -3)(2x +1) = x ( x +5)

* 2x2 +x – 6x -3 = x2 + 5x
* 2x2 – 5x -3 – x2 – 5x = 0
* x2 -10x -3 = 0 , yes it is a quadratic equation.

(iii) (x + 2)3 = 2x(x2 -1)

* x3 + 8 + 12x + 6x2 = 2x3 – 2x
* x3 + 12x + 6x2 + 8 – 2x3 + 2x = 0
* -x3 + 6x2 + 14x + 8 = 0, No it is not a quadratic equation .

Same as other parts can be solved.

**Q3. Find the values from each of these given situations:**

1. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 sq. Meters more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find its length and breadth.
2. Two water tapes together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tap.
3. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore. If the average speed of the express train is 11 km/hr. More than that of the passenger train, find the speed of the two trains.
4. Two trains leave a railway station at the same time. The first train travels due west and the second train du north. The first train travels 5 km/hr faster than the second train. If after two hours, they are 50 km apart, find the average speed of each train.
5. Two tapes together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the other to fill it, find the time in which each pipe would fill the cistern.
6. A pole has to be erected at a point on the boundary of a circular park of diameter 13 m in such a way that the difference of its distances from two diametrically opposite fixed pints A and B on the boundary is 7 m. Is it possible to do so? If yes of what distance from the two gates should be the pole erected.
7. The perimeter of a right angled triangle is five times the length of its shortest side. The area of the triangle is 15 times the length of the shortest side. Find the length of three sides of the triangle.
8. Seven years ago, Varun’s age was five times the square of the Swati’s age. Three years hence, Swati’s age will be two fifths of Varun’s age. Find their present ages.
9. The sum of the reciprocals of Rehman’s ages 3 years ago and 5 years from now is 1/3. Find his present age.
10. The age of man is twice the square of the age of his son. Eight years hence the age of man will be 4 years more than three times the age of his son. Find their present ages.

**Solution:** **(i)** Let the length of the rectangular park = x m

 Then the breadth = (x – 3) m

* Area of the rectangular park = x(x-3) = x2 – 3x
* Now the base of the isosceles triangle = (x – 3) m and altitude = 12 m
* Area of the triangle = $\frac{1}{2}$ (x -3). 12 = 6(x-3) = 6x – 18

Now A.T.Q.

Area of the rectangular park = 4 + area of the triangle

* x2 – 3x = 6x – 18 + 4 = 6x – 14
* x2 -9x +14 = 0
* x2 – (7 + 2) x + 14= 0
* x2 – 7x – 2x +14= 0
* x(x-7) -2 ( x -7) = 0
* (x -7) (x -2) = 0
* If (x -7) =0 then x = 7 and if (x -2) = 0 then x = 2
* x = 2 i.e length = 2 m not possible as breadth = length – 3
* length of th park = **7 m** and the breadth = 7 -3 = **4 m**

**(ii)**  Let the tap of smaller diameter fill the tank in x hour then the tap of larger diameter fill the tank in ( x – 10) hour.

* Part of the tank filled by the smaller diameter tap in 1 hour = $\frac{1}{x}$
* Part of the tank filled by the larger diameter tap in 1 hour = $\frac{1}{x-10}$
* Part of the tank filled by both the tapes = $\frac{1}{x}$ + $\frac{1}{x-10}$ = $\frac{x-10+x }{x(x-10)}$ = $\frac{2x-10}{x^{2}- 10x }$ ....... (1)

Now A.T.Q.

 tank filled by both the pipes in $\frac{75}{8}$ hour

* Part of the tank filled by both the pipes in 1 hour = $\frac{8}{75}$ ........... (2)
* From eq(1) and (2) we get $\frac{2x-10}{x^{2}- 10x }$ = $\frac{8}{75}$
* 75(2x – 10) = 8(x2 – 10x)
* 150x – 750 = 8x2 – 80x
* 8x2 – 230x + 750 = 0
* 4x2 – 115x + 375 = 0

Solve the equation and find the values of x .

**(iii)** Let the average speed of passenger train = x km/hr.

 Then the average speed of express train = (x + 11) km/hr.

 Distance between Mysore and Bangalor = 132 km.

 Time taken by express train = 132/(x +11) = t1

 Time taken by passenger train = 132/x = t2

 A.T.Q. t2 = t1 +1

 => $\frac{132}{x }$ = $\frac{132}{x+11 }$ +1 => $\frac{132}{x }$ - $\frac{132}{x+11 }$ = 1

 => $\frac{132\left(x+11\right)- 132 x}{x ( x+11)} $ = 1 => 132x + 1452 – 132x = x2 + 11x

 => x2 + 11x – 1452 = 0 , solve the equation to find the values .

**(iv)** Let the speed of second train = x km/hr. A

 Speed of first train = (x + 5) km/hr.

 Time taken by both the trains = 2 hrs.

 Distance travelled by first train in the East = 2( x + 5) km = BC B C

 And that of by second train in North = 2x Km. = BA

 Distance between two trains = 50 Km. = AC

 From the triangle ABC by applying PGT we get

 AC2 = AB2 + BC2

 (50)2 = (2 x)2 + (2x + 5)2 = 4x2 + 4x2 + 25 + 20 x = 8x2  + 20x + 25

 2500= 8x2 +20x + 25 => 8x2 + 20x -2475 = 0 , solve the equation to get value of x i.e speed of second train and then speed of first train .

1. The figure of the given question is

 Let P be the point at which the pole is to be erected on the A

 Boundary of the circular park, and A and B are diametrically

 Opposite points.

Let distance AP = x , then ATQ. BP = x +7 P B

AB = 13 m => In Triangle ABP we have AB2 = AP2 + BP2

* (13)2 = x2 + (x + 7)2  = x2  + x2 + 49 + 14x = 2x2 + 14x + 49
* 2x2 + 14x -120= 0 , now to check is this possible or not we have to find the discriminant i.e- b2 – 4ac = (14)2 – 4 . 2. (-120) = 196 + 960 = 1156 >0
* D > 0, hence it is possible. Now solve the equation and find x.

**(viii)** Let the present age of Varun = x and that of Shwati = y

* Seven years ago Varun’s age = x – 7 and Shwati’s age = y - 7

A.T.Q. x – 7 = 5 (y – 7)2 ................. (1)

 Again three years hence Varun’s age = x + 3 and Shwati’s age = y + 3

A.T.Q. y + 3 = $\frac{2}{5}$ ( x +3) => 5y + 15 = 2x + 6 => 2x = 5y + 15-6= 5y + 9

 => x = (5y +9)/ 2 , now putting this value of x in eq.(1) we get the equation as

 2y2 – 29y + 99 = 0 , solve the equation and find y and then x also.

**(ix)** Let the present age of Rehman = x

Age before 3 years = x – 3 and its reciprocal = $\frac{1}{x-3}$

age after 5 years = x + 5 and its reciprocal = $\frac{1}{x +5}$

A.T.Q

 $\frac{1}{x-3}$ + $\frac{1}{x+5}$ = $\frac{1}{3}$

* $\frac{x+5+x-3}{\left(x-3\right)( x+5)} $ = $\frac{1}{3}$
* 3( 2x + 2) = (x – 3) (x +5) = x2 + 5x – 3x – 15
* 6x + 6 = x2 + 2x – 15
* x2 – 4x – 21 = 0 , solve the equation and find the value of x .